Exam. Code: 0005
Sub. Code: 0443

2021

B.A./B.Sc.(General)-5th Semester

Mathematics

Paper-I: Analysis-I

Time allowed: 3 Hours | Max. Marks: 30

NOTE: *Attempt five questions in all, selecting atleast two questions from each Unit.*

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UNIT – I

I. (a) Prove that union of finite number of countable sets is a countable set.

(b) Let f be the function defined on [0,1] as $f(x)=\begin{cases}1+x & if \quad x\neq \frac{1}{2}\\ 0 & if \quad x=\frac{1}{2}\end{cases}$. Show that f is Riemann integrable on [0,1] and $\int_0^1 f(x)dx=\frac{3}{2}$. (3+3)

II. (a) Let $f(x)=x^2$ and $P=\left\{-\frac{3}{2},-\frac{1}{2},\frac{1}{4},1\right\}$. Evaluate L(P,f).

(b) State and prove Darboux's Theorem. (3+3)

III. (a) Let f be a continuous function defined on [a,b] and let $f(x)=\int_a^x f(t)dt\forall x\in[a,b]$ then prove that $\frac{d}{dx}(F(x))=f(x)\forall x\in[a,b]$.

(b) Let $f(x)=\begin{cases}1-x & if \quad x \quad is \quad irrational\\ \sqrt{1-x^2} & if \quad x \quad is \quad rational\end{cases}$. Show that f is Riemann integrable on [0,1]. (3+3)

IV. (a) Show that $\int_0^1 \frac{(1-x^4)^{\frac{3}{4}}}{(1+x^4)^2}dx=\frac{1}{4(2)^{\frac{1}{4}}}B\left(\frac{7}{4},\frac{1}{4}\right)$ where B(m,n) is the Beta function.

(b) Show that $\int_0^1 \frac{x^2}{\sqrt{1-x^4}}dx=\frac{\frac{\pi}{2\sqrt{2}}}{\int_0^\infty \frac{1}{\sqrt{1+x^4}}dx}$. (3+3)

P.T.O.

UNIT-II

V. (a) Show that $\int_0^\infty \left(\frac{1}{1+x} - e^{-x}\right)\frac{dx}{x}$ is convergent.

(b) Discuss the convergence of $\int_0^1 \left(\log\frac{1}{x}\right)^m dx$. (3+3)

VI. (a) Discuss the convergence of $\int_0^1 \frac{\left(x^m + x^{-m}\right)}{x}\log(1+x)dx$.

(b) Show that $\int_0^\infty \frac{\sin x}{x}dx$ is not absolutely convergent. (3+3)

VII. (a) Show that $\int_0^{\pi/2} \sin x \log(\sin x)dx$ is convergent with value $\log\left(\frac{2}{e}\right)$.

(b) If $|a|<|$ *Evaluate* $\int_0^\pi \frac{\log(1+a\cos x)}{\cos x}dx$ (3+3)

VIII. (a) Evaluate $\int_0^{\pi/2} \log\left(\alpha^2\cos^2\theta + \beta^2\sin^2\theta\right)$do where $\alpha > 0, \quad \beta > 0$.

(b) If f(x) is a continuous function on $(0,\infty)$ having points of infinite discontinuity at 0 and ∞ only, $\lim_{x\to 0+} f(x) = f_0$ *and* $\lim_{x\to\infty} f(x) = f_1$ then prove that $\int_0^\infty \frac{f(ax) - f(bx)}{x}dx = (f_0 - f_1)\log\frac{b}{a}$. (3+3)

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