(i) **Printed Pages: 3** **Roll No. ....**

(ii) **Questions : 8** **Sub. Code : 0 5 4 1**

Exam. Code : 0 0 0 6

B.A./B.Sc. (General) 6th Semester

(2042)

MATHEMATICS

Paper : I (Analysis–II)

Time Allowed : Three Hours] **[Maximum Marks : 30**

Note :— Attempt **FIVE** questions in all, selecting at least **TWO** questions from each unit.

UNIT—I

1\. (a) Let $A = \{(x, y) \mid 1 \leq x \leq 3, 2 \leq y \leq 4\}$. Define

$$f : A \to \mathbb{R} \text{ as } f(x, y) = \begin{cases} 2 & \text{if } x \text{ is rational} \\ -2 & \text{if } x \text{ is irrational} \end{cases}$$

Use definition of double integral to show that f(x, y) is not integrable over A.

(b) Evaluate $\iint_A xy\, dA$, where A is the region common to the circles $x^2 = y^2 = 2x$; $x^2 + y^2 = 2y$. 3+3=6

2\. (a) Find the area enclosed by $4x^2 + y^2 = 36$ using double integration.

(b) Find the volume of a truncated cone with end radii 7 and 3 and height 10; using Triple integral. 3+3=6

3. (a) Evaluate the integral $\int_0^1 \int_0^{\sqrt{1-x^2}} \frac{1}{(1+e^y)\sqrt{1-x^2-y^2}} dy\, dx$.

(b) Evaluate using Green's Theorem in plane for

$$\oint_C [(\cos x \sin y - xy)dx + \sin x \cos y \, dy]$$

where C is the circle $x^2 + y^2 = 1$. 3+3=6

4. (a) Verify Gauss Divergence Theorem for the vector $\vec{A} = x\hat{i} + y\hat{j} + z\hat{k}$ over the region bounded by $x^2 + y^2 + z^2 = 25$.

(b) Verify Stoke's Theorem for $\vec{F} = (y - \sin x)\hat{i} + \cos x\hat{j}$ over the triangle with vertices (0, 0), $\left(\frac{\pi}{2}, 0\right)$, $\left(\frac{\pi}{2}, 1\right)$.

3+3=6

UNIT—II

5. (a) Prove that product of two uniformly convergent sequences need not be uniformly convergent.

(b) Use Weierstrass's M-test to show that the series $\sum_{n=1}^{\infty} \frac{a_n}{1+x^{2n}}$ converges uniformly $\forall x \in \mathbb{R}$, if Σa_n is absolutely convergent. 3+3=6

6. (a) Show that the sum function of a uniformly convergent series of continuous functions is itself continuous.

(b) Prove that the series $\sum_{n=1}^{\infty}\left[\frac{nx}{1+n^2x^2}-\frac{(n-1)x}{1+(n-1)^2x^2}\right]$ can be integrated term by term on [0, 1] although it is not uniformly convergent on [0, 1]. 3+3=6

7. (a) Prove that $\tan^{-1}x = x-\frac{x^3}{3}+\frac{x^5}{5}-\frac{x^7}{7}+.....$ for $-1 \le x \le 1$.

(b) Show that for $-\pi < x < \pi$,

$$\frac{x(\pi^2-x^2)}{12}=\frac{\sin x}{1^3}-\frac{\sin 2x}{2^3}+\frac{\sin 3x}{3^3}-\frac{\sin 4x}{4^3}+.....$$

3+3=6

8. (a) Find the Fourier expansion for the function $f(x) = x - x^3$ in the interval $-1 < x < 1$.

(b) Express f(x) as cosine series on $[0, \pi]$, where

$$f(x)=\begin{cases}\pi/3 & 0\le x\le \pi/3\\ 0 & \pi/3\le x\le 2\pi/3\\ -\pi/3 & 2\pi/3\le x\le \pi\end{cases}$$

3+3=6