

2056
Bachelor of Arts (FYUP) Second Semester
Mathematics
Paper: Calculus - I (Old)
(Common with B.Sc. 2nd Semester FYUP)

Time allowed: 3 Hours

Max. Marks: 90

NOTE: Attempt five questions in all, including Question No. 1 which is compulsory and selecting one question from each Unit.

x-x-x

- I. Attempt the following:-
a) State trichotomy law of real numbers.

b) Evaluate $\lim_{x \rightarrow 0} \frac{1 - \cos x}{2x}$.

c) Find $\frac{dy}{dx}$ where $y = \tanh(1 + x^2)$.

d) Find third derivative of $\log \sqrt{3x+4}$ w.r.t x.

- e) State Rolle's theorem

f) $\lim_{x \rightarrow 0} \frac{\sin ax}{\sin bx} = \frac{a}{b}$, prove or disprove. (6x3)

UNIT - I

- II. a) Prove that $\sqrt{3}$ is irrational number.
b) State and prove Archimedean property of real numbers.
c) Solve the equation $|x^2 + 10x + 8| = |x^2 + 6x + 15| + |4x - 7|$. (3x6)

- III. a) Prove that; if $\lim_{x \rightarrow a} f(x)$ exists finitely, then it is unique.
b) If $f(x) = \begin{cases} 0 & \text{if } x \text{ is irrational} \\ 1 & \text{if } x \text{ is rational} \end{cases}$ show that $\lim_{x \rightarrow c} f(x)$ does not exist for any $c \in \mathbb{R}$. (2x9)

UNIT - II

- IV. a) If a function f is continuous at a point a and $f(a) \neq 0$ then prove that there exists some neighborhood of a where f(x) has the same sign as that of f(a).
b) Examine the continuity of the function

$$f(x) = \begin{cases} \frac{x-a}{|x-a|} & \text{if } x \neq a \\ 1 & \text{if } x = a \end{cases} \quad \text{at } x = a. \quad (2 \times 9)$$

P.T.O.

V. a) Let $f(x) = \begin{cases} 1 & \text{if } x \leq 3 \\ ax+b & \text{if } 3 < x < 5 \\ 7 & \text{if } 5 \leq x \end{cases}$

Find values of a and b so that f may be continuous for all x.

b) Show that $f(x) = \begin{cases} x \sin \frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$ is continuous for all real x. (2x9)

UNIT - III

VI. a) Find $\frac{d}{dx} \{x^x + (\cosh x)^x\}$

b) Find $y_n(0)$ where $y = (\sin^{-1} x)^2$. (2x9)

VII. a) Find a, b and c, so that $\lim_{x \rightarrow 0} \frac{a \sin x - bx + cx^2 + x^3}{2x^2 \log(1+x) - 2x^3 + x^4}$ is finite and also find the limit.

b) Use differentials, find approximate value of $\sqrt{290}$.

c) Find the equation of normal to the curve $y^2 = 4x$ at the point (1,2). (9,5,4)

UNIT - IV

VIII. a) State and prove Cauchy's mean value theorem.

b) Find first three non zero terms in the Maclaurin's expansion of $\sin x$. (2x9)

IX. a) State and prove L.M.V. theorem.

b) Discuss the applicability of Rolle's theorem for $f(x) = |x|$ in $-1 \leq x \leq 1$.

c) Express $2x^3 + 7x^2 + x - 6$ in powers of $x-2$. (9,5,4)

x-x-x