(i) Printed Pages : 4 Roll No. .....

(ii) Questions : 9 Sub. Code : 1 1 2 5 6

Exam. Code : 5 0 0 4

Bachelor of Arts (FYUP) 4th Semester

(2056)

MATHEMATICS

Paper : Differential Equations-I

(Common with B.Sc. 4th Sem. FYUP)

Time Allowed : 3 Hours] **[Maximum Marks : 80**

Note :— Attempt **five** questions in all, selecting **one** question from each unit and the compulsory question. Each question will carry equal marks.

(Compulsory Question)

1. (i) Let $f(x,y)=\log(xy+2y^2-2x)$. Find $fx(2,3)$ and $fy(2,3)$.

(ii) Solve the Clairaut's equation $y = xy' + \frac{1}{y'}$

(iii) Find the orthogonal trajectory of the curve $x^2y = c$

(iv) Solve the following differential equation:

$$x^2\frac{d^2y}{dx^2}-3x\frac{dy}{dx}+3y=x^2.$$ 4×4=16

Scanned with OKEN Scanner

UNIT—I

2. (i) Let $f : R^2 \to R$ be defined by :

$$f(x,y)=\begin{cases}\dfrac{xy^2}{x^2+y^2} & \text{for}(x,y)\neq(0,0)\\ 0 & \text{for}(x,y)=(0,0)\end{cases}$$

Discuss the continuity of f at (0,0). 8

(ii) By using definition, prove that $\lim\limits_{(x,y)\to(1,2)}\left(x^2+y^2\right)=5.$ 8

3. (i) If $u = \log (x^3 + y^3 + z^3 - 3xyz)$, show that :

$$\left(\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\frac{\partial}{\partial z}\right)^2 . u=-\frac{9}{(x+y+z)^2}.$$ 8

(ii) State and prove Euler's theorem on homogeneous functions of three variables. 8

UNIT—II

4. (i) Solve $(3x^2y^4 + 2xy)dx + (2x^3y^3 - x^2)dy = 0.$ 8

(ii) Solve :

$$p=\tan\left(x-\frac{P}{1+p^2}\right).$$ 8

Scanned with OKEN Scanner

5. (i) Solve : $x^2 (y - px) = p^2y$. 8

(ii) Solve $y + px = x^4p^2$ and find the singular solution and explain the geometrical significance of the irrelevant factors that present themselves. 8

UNIT—III

6. (i) Find the orthogonal trajectory of $r^n = a^n \cos n\theta$. 8

(ii) Solve :

$$\frac{d^4y}{dx^4} - \frac{d^2y}{dx^2} = 4$$ 8

7. (i) Solve :

$$\frac{d^4y}{dx^4} - y = x^2 \sin x$$ 10

(ii) Solve the differential equation:

$$\left(0^2 + 36\right) y = \sin 6x$$ 6

UNIT—IV

8. (i) Solve the differential equation:

$$x^3 \frac{d^3y}{dx^3} + 6x^2 \frac{d^2y}{dx^2} + 4x \frac{dy}{dx} - 4y = (\log x)^2$$ 8

Scanned with OKEN Scanner

(ii) Solve the differential equation:

$$(5+2x)^2\frac{d^2y}{dx^2}-6(5+2x)\frac{dy}{dx}+8y=0$$ 8

9. (i) Solve the following differential equation:

$$\left(x^2D^2+3xD+1\right)y=(1-x)^{-2}$$ 10

(ii) Solve :

$$\left[\left(x^3-4x\right)D^3+\left(9x^2-12\right)D^2+18xD+6\right]y=0$$ 6

Scanned with OKEN Scanner