(i) Printed Pages : 4 Roll No. ....

(ii) Questions : 9 **Sub. Code : 1 1 2 5 3**

Exam. Code : 5 0 0 4

Bachelor of Arts (FYUP) 4th Semester

(2056)

MATHEMATICS

Paper : Discrete Mathematics & Graph Theory
(Common with B.Sc. 4th Sem. FYUP)

Time Allowed : 3 Hours] [Maximum Marks : 80

Note :– Attempt **five** questions in all, selecting **one** question from each Unit and the compulsory question number **1**. Each question will carry marks as indicated.

UNIT–I

1. (a) Write the dual of the compound proposition $(p \wedge \sim q) \vee (q \wedge F)$. 4

(b) Consider the divisibility relation on the set {3,5,9,15,24,45}. Find the least upper bound and greatest lower bound of {3,5}, if they exist. 4

(c) Draw the Kuratowski's graphs $k_{3,3}$ and k_5. 4

(d) Show that $f(x) = x^2 + 2x + 1$ is $O(x^2)$. 4

UNIT–II

2. (a) Show that $p \vee (q \wedge r)$ and $(p \vee q) \wedge (p \vee r)$ are logically equivalent. 8

(b) By constructing a truth table, show that the Boolean expression $p \vee \sim (p \wedge q)$ is a tautology. 8

3. (a) Let p and q be the propositions "The election is decided" and "The votes have been counted" respectively. Express each of these compound propositions as an English sentence.

(i) $\sim p$

(ii) $\sim p \wedge q$

(iii) $\sim q \rightarrow \sim p$

(iv) $p \leftrightarrow q$ 8

(b) Let the universe U be the set of all human beings living in the year 2026, and let P(x) stand for "x is young," Q(x) for "x is female," and R(x) for "x is an athlete." Translate the following English sentences into quantified formulas:

(i) Some athletes are female and are not young.

(ii) Some young people are not athletes. 8

UNIT–III

4. (a) Let $A = \{a, b, c\}$ be a given finite set and $P(A)$ its power set. Prove that $P(A)$ forms a poset with inclusion relation $\subseteq$ and draw its HASSE diagram. 8

(b) Let D_n denote the set of all positive divisors of the integer n. Prove that the set D_{90}, under the divisibility relation, forms a Lattice. 8

5. (a) Using Boolean algebra, show that

$$abc + \bar{a}bc + a\bar{b}c + ab\bar{c} = ab + ac + bc. \quad 8$$

(b) Prove that, the complement of every element in a Boolean algebra B is unique. 8

UNIT–IV

6. (a) Define the following terms in Graph Theory: Walk, Path, Circuit, and Tree. 8

(b) Let G be a connected planar simple graph with E edges and V vertices. Let R be the number of regions in a planar representation of G. Then prove that

$$V - E + R = 2 \quad 8$$

7. (a) If a graph G has more than two vertices of odd degree, then prove that there can be no Euler path in G. 8

(b) Prove that a simple graph is connected if and only if it has a spanning tree. 8

UNIT–V

8. (a) Find the domain and range of the function $(f \circ g)(x)$ where f and g are real valued functions defined by $f(x) = \sqrt{x}$ and $g(x) = x^2 - 5x + 6$. 8

(b) Solve the recurrence relation $s_n - 7s_{n-1} + 10s_{n-2} = 6 + 8n$ with $s_0 = 1$, $s_1 = 2$. 8

9. (a) Differentiate between error detection and error correction techniques giving one example in each. 8

(b) Define a Characteristic and a hashing function in Computer Science, and give an example in each. 8