

2036  
Bachelor of Arts (FYUP) Fourth Semester  
Mathematics  
Paper: Coordinate Geometry  
(Common with B.Sc. 4<sup>th</sup> Semester FYUP)

Time allowed: 3 Hours

Max. Marks: 80

*NOTE: Attempt five questions in all, including Question No. 1 which is compulsory and selecting one question from each Unit.*

$x-x-x$

1. Attempt the following:-

- If the axes be turned through an angle of  $\tan^{-1} 2$ , how does the equation  $11x^2 + 14y^2 - 4xy = 5$  change?
- For what value of  $k$  will the line  $y = 2x + k$  be tangent to the circle  $x^2 + y^2 - 5 = 0$
- Find the equation of tangents to the ellipse  $\frac{x^2}{16} + y^2 = 1$  which make an angle of  $60^\circ$ .
- Classify the conic  $x^2 - 4xy + 4y^2 - 32x + 4y + 16 = 0$ . (4x4)

UNIT - I

- By suitable transformation of axes, transform the equation  $x^2 + 4xy + y^2 - 3x = 6$  into an equation that has no product term.
  - For what values of  $K$  does the equation  $3x^2 + 9xy + Ky^2 + 3x + 3y + 3 = 0$  represents two straight lines? Are these real or imaginary? (2x8)
- Find the area of the triangle bounded by the lines  $y^2 - 9xy + 18x^2 = 0$  and  $y = 9$ .
  - Show that the equation  $9x^2 - 24xy + 16y^2 - 12x + 16y - 12 = 0$  represents a pair of parallel straight lines. Find the distance between them. (2x8)

UNIT - II

- Find the angle between the tangents from the point (3,4) to the circle  $x^2 + y^2 - 2x + 4y = 0$ .
  - The line  $3x + 4y = 25$  cuts the circle  $x^2 + y^2 - 50 = 0$  at two points P and Q. The tangents at P and Q meet at R. Find the point R. (2x8)
- Find the locus of the middle points of the chords of the circle  $x^2 + y^2 + 6x + 2y - 10 = 0$  which subtend a right angle at the centre of the circle.
  - Find the equation of the circle which passes through (1,1) and cuts orthogonally each of the circles  $x^2 + y^2 - 8x - 2y + 16 = 0$  and  $x^2 + y^2 - 4x - 4y - 1 = 0$ . (2x8)

P.T.O.



(2)

**UNIT - III**

6. a) A tangent to the parabola  $y^2 + 4bx = 0$  meets the parabola  $y^2 = 4ax$  in P and Q. Show that the locus of the middle point of PQ is  $y^2(2a + b) = 4a^2x$ .
- b) Show that the joint equation of the pair of tangents drawn to the ellipse  $\frac{x^2}{16} + \frac{y^2}{9} = 1$  from the point (4,3) is  $xy - 3x - 4y + 12 = 0$ . Also prove that they are at right angles. (2x8)
7. a) Show that the locus of the poles of the normal chords of the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  is the curve  $\frac{a^6}{x^2} + \frac{b^6}{y^2} = (a^2 - b^2)^2$ .
- b) Show that the locus of the middle points of the chords of the hyperbola  $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$  whose pole lies on the line  $x + y - 1 = 0$  is the hyperbola  $\frac{x^2}{a^2} - \frac{y^2}{b^2} = x + y$ . (2x8)

**UNIT - IV**

8. a) If  $y + x = 0$  and  $2y - x = 0$  are a pair of conjugate diameters of the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ , find its eccentricity.
- b) Prove that the locus of the mid-points of a system of parallel chords of the hyperbola  $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$  is a straight line passing through the centre of the hyperbola. (2x8)
9. a) By using suitable transformation of the axes reduce  $3x^2 + 8xy - 3y^2 - 40x - 20y + 50 = 0$  to standard form and hence identify the conic.
- b) Find the equation of the asymptotes to the hyperbola  $3x^2 - 5xy - 2y^2 + 5x + 11y - 8 = 0$ . (2x8)

x-x-x