(i) Printed Pages : 3 Roll No. .....

(ii) Questions : 8 Sub. Code : 1 7 5 4 2

Exam. Code : 0 0 0 6

B.A./B.Sc. (General) 6th Semester

(2056)

MATHEMATICS

Paper—II : Linear Algebra

Time Allowed : 3 Hours] **[Maximum Marks : 30**

Note :– Attempt **five** questions in all, selecting at least **two** from each Unit.

UNIT—I

1. (a) If W_1 and W_2 are subspaces of vector space V(F), prove that $W_1 + W_2$ is also a subspace of V(F). 3

   (b) Let V(R) be a vector space of all functions from R to R. Then show that

   $W = \{f \mid f \in V \text{ and } f(-x) = f(x)\}$ is subspace of V. 3

2. (a) Write (3, 2, 1) as a linear combination of vectors $x_1 = (2, -1, 0)$, $x_2 = (1, 2, 1)$, and $x_3 = (0, 2, -1)$. 3

   (b) Find the value of k so that the vectors (1, –1, 3), (1, 2, –2), and (k, 0, 1) are L.D. 3

3. (a) Prove that any two bases of a finite-dimensional vector space V(F) have the same number of elements. 3

(b) Extend $\{(1, 2, 3)\}$ to two different bases of $V_3(R)$. 3

4. (a) Find a linear transformation $T : R^2 \to R^2$ s.t.

$T(1, 0) = (1, 1)$ and $T(0, 1) = (-1, 2)$. 3

(b) State and prove the Rank-Nullity theorem. 3

UNIT—II

5. Let T be a linear operator defined by $T(x, y) = (4x - 2y, 2x + y)$ and $B = \{(1, 1), (-1, 0)\}$ be a basis of R^2. Show that

$[T : B]\ [v\ B] = [T(v, B]\ \forall\ v \in R^2$. 6

6. (a) Consider the bases $B_1 = \{(1, 0), (0, 1)\}$ and $B_2 = \{(1, 2), (2, 3)\}$ of R^2. Find the transition matrix from B_1 to B_2. 3

(b) State and prove the Cayley-Hamilton theorem. 3

7. If the matrix of a linear operator T relative to the usual basis of R^3 is $\begin{bmatrix} 1 & 1 & -1 \\ -1 & 1 & 1 \\ 1 & -1 & 1 \end{bmatrix}$, find its matrix relative to the basis

$B_1 = \{(1, 2, 2), (1, 1, 2), (1, 2, 1)\}$. 6

8. (a) Show that if λ is an eigenvalue of matrix A over field F, then λ^2 is an eigenvalue of A^2. 3

(b) Find the minimal polynomial of $\begin{bmatrix} 5 & -6 & -6 \\ -1 & 4 & 2 \\ 3 & -6 & -4 \end{bmatrix}$. 3