(i) Printed Pages : 4 Roll No. .....

(ii) Questions : 9 Sub. Code : 2 6 1 0 1

Exam. Code : 0 4 7 2

M.Sc. Physics 1st Semester

(2125)

MATHEMATICAL PHYSICS–I

Paper : PHY-8011

Time Allowed : Three Hours] **[Maximum Marks : 60**

Note :— (1) Attempt **FIVE** questions in all, taking **ONE** question each from Units I to IV.

(2) Unit-V is compulsory.

UNIT–I

I. (a) Obtain the necessary and sufficient conditions for a function to be analytic. 6

(b) Using the calculus of residues, evaluate

$$\int_0^{2\pi} \frac{\cos 2\theta}{5-4\cos\theta} d\theta.$$ 6

II. (a) State and obtain expression for Laurent series. Hence define the analytic and principal part of the series. 6

(b) Solve the integral by method of residues

$$\int_{-\infty}^{\infty} \frac{\cos 3x}{(x^2+1)(x^2+4)} dx.$$ 6

UNIT–II

III. (a) Evaluate $I = \int_0^\infty \frac{x^n}{a^x} dx$ using beta gamma functions. 4

(b) Define Dirac delta function. Show that the function $\frac{1}{2\pi}\int_{-\infty}^{\infty} e^{ikx} dk$ satisfies all properties of Dirac delta function. 4

(c) Find the value of $\left(\frac{1}{2}\right)$ using two methods. 4

IV. (a) Using duplication formula

$$\Gamma(m)\,\Gamma\left(m+\frac{1}{2}\right) = \Gamma(2m)\,\frac{(\pi)^{\frac{1}{2}}}{2^{2m-1}}.$$

Hence show that $\beta(m, m) = 2^{1-2m}\,\beta\left(m, \frac{1}{2}\right)$. 4

(b) Prove that $\int_{-\infty}^{\infty} \delta(a-y)\,\delta(y-b) = \delta(a-b)$. 4

(c) Evaluate $\int_0^{\frac{\pi}{2}} \frac{d\theta}{\sqrt{1-\frac{\sin^2\theta}{2}}}$. 4

UNIT–III

V. (a) Solve the $3xy''(x) + 2y'(x) + y(x) = 0$ using Frobenius method. 6

(b) Using the method of separation of variables, solve Laplace equation in spherical polar coordinates. 6

VI. (a) Determine the steady state temperature distribution in a thin plate bounded by the lines $x = 0$, $x = 1$, $y = 0$ and $y = \infty$; assuming that heat can't escape from either surface of the plate, the edges $x = 0$, $x = 1$, $y = \infty$, being kept at steady temperature $F(x)$. 6

(b) Find the eigenvalues and normalized eigenvectors of the following matrix:

$$\begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix}.$$ 6

UNIT–IV

VII. (a) Obtain the orthonormal property of Hermite polynomial. 6

(b) Obtain the Rodrigue's formula for Laguerre polynomials. Hence find $L_3(x)$. 6

VIII. (a) Deduce the orthogonality relation for Legendre polynomials. 6

(b) Obtain the expression for generating function of Bessel function. 6

UNIT–V
(Compulsory Question)

IX. (a) Show that: $x J_n'(x) = n J_n(x) + x J_{n+1}(x)$. 2

(b) Find the analytic function $f(z) = u(x, y) + iv(x, y)$ if $u(x, y) = x^3 - 3xy^2$. 2

(c) Evaluate $\oint_C \frac{\sin^2 z\, dz}{(z-a)^4}$. 2

(d) Show that every square matrix can be uniquely expressed as the sum of a symmetric and a skew-symmetric matrix. 2

(e) Give the physical meaning of curl of a vector. 2

(f) Write Schrödinger equation in Cartesian and spherical polar coordinates in three dimensions. 2