

Time allowed: 3 Hours

Max. Marks: 60

NOTE: Attempt five questions in all, including Question No. 1 which is compulsory and selecting atleast one question from each Unit.

x-x-x

I. Attempt the following:-

- (a) Simplify $\frac{\sqrt{18} + \sqrt{8}}{\sqrt{2}}$ and $(27)^{\frac{2}{3}} \times (9)^{-\frac{1}{2}}$
- (b) Express $\frac{3+4i}{4+3i}$ in the form of $a + ib$. Find its conjugate and modulus.
- (c) Show that a relation R on the set of integers given by $R = \{(a, b) : a - b \text{ is an even number}\}$ is an equivalence relation.
- (d) Find the maximum and minimum values of $f(x) = x^2 - 4x + 5$.
- (e) What is the order and degree of the differential equation $\frac{d^2y}{dx^2} + 5\left(\frac{dy}{dx}\right)^3 + 9y = 0$.
- (f) How many matrices of order 3×3 can be constructed with each entry either 0 or 1. (6×2)

UNIT - I

- II.** (a) Show that $\sqrt{3} + \sqrt{5}$ is an irrational number.
- (b) For any complex number z show that $z \cdot \bar{z} = |z|^2$ and compute \sqrt{i} . (2×6)
- III.** (a) Let U be a set having 100 elements. If X and Y are two subsets such that $(X \cup Y)^c$ has 60 elements, X has 28 elements and Y has 32 elements, how many elements does $X \cap Y$ have?
- (b) Find a if the 17th and 18th terms of the expansion $(2 + a)^{50}$ are equal. (2×6)

UNIT - II

- IV.** (a) Prove that the function $f(x) = \begin{cases} \frac{x}{|x| - 2x}, & x \neq 0 \\ k, & x = 0 \end{cases}$ is discontinuous at $x = 0$, regardless the choice of k .
- (b) Find $\frac{dy}{dx}$ for (i) $y = \sqrt{\frac{4+x}{4-x}}$, and (ii) $y = (\sin \sqrt{x})^3$ (2×6)

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(2)

- V. (a) Find the intervals in which the function $f(x) = x^3 - 6x^2 + 9x + 1$ is increasing and decreasing.
- (b) Evaluate $\int_0^{\frac{\pi}{2}} \sin^2 x \, dx$ (2×6)

UNIT - III

- VI. (a) Without expanding the determinants show that $\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} + \begin{vmatrix} 1 & 1 & 1 \\ yz & zx & xy \\ x & y & z \end{vmatrix} = 0$

- (b) Solve the following system of equations using matrices:

$$\begin{aligned} x + y + z &= 6, \\ 2x - y + 3z &= 14, \\ x + 2y - z &= 2. \end{aligned} \quad (2 \times 6)$$

- VII. (a) Let p = Maria learns mathematics and q = Maria finds a good job, be two statements. Write down converse, reverse and contra-positive statements, in English language, of the statement $p \rightarrow q$.
- (b) Convert the following:
- (i) $(1101)_2$ to decimal and hexadecimal form.
- (ii) $(357)_8$ to binary form. (2×6)