(i) Printed Pages : 3 Roll No. .....

(ii) Questions : 9 Sub. Code : 1 0 8 2 0

Exam. Code : 5 0 0 3

Bachelor of Arts (FYUP) 3rd Semester Examination

(2125)

MATHEMATICS

Paper–Differential Equations-I

(Common With B.Sc. 3rd Sem. NEP)

Time Allowed : Three Hours] **[Maximum Marks : 80**

Note :— Attempt ***five*** questions in all, selecting ***one*** question from each unit and the compulsory question. Each question will carry equal marks.

(Compulsory Question)

1. (i) Prove that $\lim\limits_{\substack{x\to 0\\ y\to 0}} \dfrac{x^4y^4}{(x^2+y^2)^3}$ does not exist.

(ii) Solve the following differential equation :

$(5x^4 + 3x^2y^2 - 2xy^3)dx + (2x^3y - 3x^2y^2 - 5y^4)\, dy = 0$

(iii) Find the orthogonal trajectory of the family of parabolas $y = ax^2$.

(iv) Solve the following differential equation :

$$x^2\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + 3y = x^2.$$ 4×4

UNIT—I

2. (i) Examine the function $f(x,y)=\frac{xy^3}{x^2+y^2}$, $x \neq 0$, $y \neq 0$ $f(0, 0) = 0$ for continuity at (0, 0). 8

(ii) Let $f(x,y)=y\sin\frac{1}{x}+x\sin\frac{1}{y}$, where $x \neq 0$, $y \neq 0$. Prove that $f(x, y) \to 0$ as $(x, y) \to (0, 0)$. 8

3. (i) If $u = \log (x^3 + y^3 + z^3 - 3xyz)$, show that

$$\left(\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}\right)^2=\frac{9}{(x+y+z)^2}.$$ 8

(ii) State and prove Euler's theorem on homogeneous functions of three variables. 8

UNIT—II

4. (i) Solve : $x^2ydx - (x^3 + y^3)\, dy = 0$. 8

(ii) Solve : $y - 2px = \tan^{-1}(xp^2)$. 8

5. (i) Solve $(px - y)(py + x) = h^2p$. 8

(ii) Obtain the primitive and singular solution, if it exists, of the equation $xp^2 - 2yp + 4x = 0$. 8

UNIT—III

6. (i) Find the orthogonal trajectory of $r^n = a^n \cos n\theta$. 8

(ii) Solve : $(D^2 + 2D + 1)\, y = e^{-x}$. 8

7. (i) $\frac{d^4y}{dx^4} - y = x^2 \sin x.$ 10

(ii) Solve the differential equation : $(D^2 + 36)y = \sin 6x.$ 6

UNIT—IV

8. (i) Solve the differential equation :

$$x^3 \frac{d^3y}{dx^3} + 3x^2 \frac{d^2y}{dx} + x \frac{dy}{dx} + y = x \log x.$$ 8

(ii) $(2x-1)^3 \frac{d^3y}{dx^3} + (2x-1)\frac{dy}{dx} - 2y = 0.$ 8

9. (i) Solve the differential equation :

$$x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} - y = x^2 e^x.$$ 10

(ii) Solve :

$[(x^3 - 4x) D^3 + (9x^2 - 12) D^2 + 18x D + 6]y = 0.$ 6