(i) Printed Pages : 2 Roll No. ....

(ii) Questions : 8 **Sub. Code : 1 7 4 4 4**

Exam. Code : 0 0 0 5

B.A./B.Sc. (General) 5th Semester

(2125)

MATHEMATICS

Paper–II : Modern Algebra

Time Allowed : Three Hours] **[Maximum Marks : 30**

Note :—Attempt **FIVE** questions in all by selecting at least **TWO** from each unit.

UNIT—I

1. (a) Define the following :
   (i) Semi group
   (ii) Normal subgroup
   (iii) Centre of a group. 3

   (b) If in a group G, $a^5 = e$, where e is identity of G and $aba^{-1} = b^2 \ \forall\ a, b \in G$. Prove that if $b \neq e$ then $o(b) = 31$. 3

2. (a) If H and K are finite subgroups of a group G, then show that $o(HK) = \dfrac{o(H) \cdot o(K)}{o(H \cap K)}$. 3

   (b) Show that the centre of a group G is a normal subgroup of G. 3

3. (a) If f: $G_1 \rightarrow G_2$ is a group homomorphism, then show that Ker f is a normal subgroup of G_1. 3

(b) Show that the converse of Lagrange's theorem is true for Abelian groups. 3

4. (a) Show that every group of order 9 is Abelian. 3

(b) Show that the order of the centre of S_3 is one. 3

UNIT—II

5. (a) Define the following:

(i) Sub-ring

(ii) Skew Field

(iii) Prime ideal. 3

(b) Show that there does not exist an I.D. with six elements. 3

6. (a) Show that {0} is a prime ideal of Z but not a maximal ideal. 3

(b) Show that every finite I.D. is a field. 3

7. (a) If f : $R_1 \rightarrow R_2$ is a ring homomorphism, then show that Ker f is an ideal of R_1. 3

(b) Find the group of units of Z [i] . 3

8. If R is a commutative ring with unity, then show that M is a maximal ideal of R iff R/M is a field. 6