

26/11/25
(M)

(i) Printed Pages : 2

Roll No.

(ii) Questions : 8

Sub. Code :

1	7	4	4	3
---	---	---	---	---

Exam. Code :

0	0	0	5
---	---	---	---

B.A./B.Sc. (General) 5th Semester

(2125)

MATHEMATICS

Paper-I : Analysis-I

Time Allowed : Three Hours]

[Maximum Marks : 30

Note :— Attempt FIVE questions in all, selecting at least TWO questions from each unit.

UNIT-I

1. (a) Prove that the set of even integers is equivalent to the set of odd integers.

(b) Evaluate $L(P, f)$ where $f(x) = x^2$ and $P = \{-5/2, -1/2, 3/2, 1\}$. 3+3=6

2. (a) Prove that every continuous defined on $[2, 5]$ is Riemann integrable.

(b) Evaluate $\int_0^3 |x^2 + 2x + 3| dx$. 3+3=6

3. (a) Prove that $\frac{3}{4} \leq \int_1^4 \frac{x}{x+1} dx \leq \frac{4}{5}$.

(b) Show that $\int_0^{\infty} \frac{x^3}{(1+x)^7} dx = \frac{1}{60}$ (using Beta function).

3+3=6

4. State and prove Duplication formula.

6

UNIT-II

5. Prove that $\int_a^b \frac{dx}{(x-a)^p}$ convergent iff $p < 1$.

6

6. (a) Test the convergence of the integral $\int_1^2 \frac{x}{\sqrt{x-1}} dx$.

(b) Discuss the convergence of the integral $\int_0^{\pi/2} \frac{\cos x}{x^2} dx$.

3+3=6

7. (a) Using Frullani's integral, prove that

$$\int_0^{\infty} \frac{e^{-ax} - e^{-bx}}{x} dx = \log\left(\frac{b}{a}\right); a > 0, b > 0.$$

(b) Prove that $\int_0^{\infty} \frac{e^{-kx} \sin x}{x} dx = \cot^{-1}(k)$ for $k > 0$.

3+3=6

8. Prove that $\int_0^{\pi/2} \frac{\log(1+b \sin^2 x)}{\sin^2 x} dx = \pi(\sqrt{1+b} - 1)$.

6