Exam Code: 0001
Sub. Code: 17045

2125
B.A./B.Sc. (General) First Semester
Mathematics
Paper – III: Trigonometry and Matrices

Time allowed: 3 Hours

Max. Marks: 30

NOTE: *Attempt five questions in all, selecting atleast two questions from each Unit.*

x-x-x

UNIT – I

I. a) Use De Moivre's theorem to solve $(4+x)^5-(4-x)^5=0$.

b) If $\tan(x+iy)=\cosh(\alpha+i\beta)$ prove that $\tanh\alpha\tan\beta = cosec2x\ sinh2y$. (2x3)

II. a) Find the product $(x-\xi)\ (x-\xi^3)\ (x-\xi)^5\ (x-\xi)^7$, ξ is a primitive 8^{th} root of unity.

b) Find all the values of $\left(1+\sqrt{3i}\right)^{3/4}$ and find their continued product. (2x3)

III. a) Using Gregory series, prove: $1+\frac{1}{3}-\frac{1}{5}-\frac{1}{7}+.........=\frac{\pi}{2\sqrt{2}}$.

b) Sum the series upto infinite series:-

$$\sin\alpha-\frac{\sin(\alpha+2\beta)}{\lfloor 2}+\frac{\sin(\alpha+4\beta)}{\lfloor 4}+.........\infty$$ (2x3)

IV. a) If $i^{i^{i\cdots\infty}}=A+i\beta$ and only principal value are considered prove that:-

i) $\tan\left(\frac{\pi A}{2}\right)=\frac{B}{A}$

ii) $A^2+B^2=e^{-AB}$

b) Prove that $\lim_{x\to 0}\frac{1}{x^2}\log\left(\frac{\tan^{-1}x}{x}\right)=\frac{-1}{3}$. (2x3)

P.T.O.

UNIT – II

V. a) Prove that $B'AB$ is symmetric and skew symmetric according as A is symmetric or skew symmetric.

b) Show that every square matrix can be uniquely expressed as sum of symmetric and skew symmetric matrix. (2x3)

VI. a) State and prove Cayley-Hamilton theorem.

b) Prove that characteristic roots of a unity matrix are unit modulus. (2x3)

VII. a) Find the root of the matrix $\begin{bmatrix} 9 & 0 & 2 & 3 \\ 0 & 1 & 5 & 6 \\ 4 & 5 & 3 & 0 \end{bmatrix}$ by reducing it to normal form.

b) Express the following matrix as the sum of a Hermitian and Skew Hermitian matrix.

$$\begin{bmatrix} 2-i & 3 & 1+i \\ -5 & 0 & -6i \\ 7 & i & -3+2i \end{bmatrix}$$ (2x3)

VIII. Find values of λ and μ for which the system of equations:

$$x + y + z = 6$$

$$x + 2y + 3z = 10$$

$$x + 2y + \lambda z = \mu$$

a) Has no solution

b) Infinitely many solutions

Also find solutions for these values of K. (6)

x-x-x