(i) Printed Pages : 4 Roll No. ....

(ii) Questions : 9 **Sub. Code : 1 0 0 0 2**

Exam. Code : 5 0 0 1

Bachelor of Arts (FYUP) 1st Semester

(2125)

MATHEMATICS

Paper : Algebra & Trigonometry MATDSC1

(Common With B.Sc. 1st Sem. N.E.P.)

Time Allowed : Three Hours] **[Maximum Marks : 80**

Note :— Attempt **five** questions in all, selecting **one** question from each unit. Question No. **1** is compulsory. Each question carries equal marks.

I. (a) Solve the equation : $\sin 2x + \sin 4x + \sin 6x = 0$ 4

(b) Prove that $i \log\left(\frac{x-i}{x+i}\right) = \pi - 2\tan^{-1} x, x > 0.$ 4

(c) Show that the vectors $x = (1, 2, 1)$, $y = (2, 1, 4)$, $z = (4, 5, 6)$ are linearly independent. 4

(d) Prove that an eigenvector of a matrix cannot correspond to two distinct eigen values. 4

UNIT—I

II. (a) Prove that $1+\frac{1}{4}+\frac{1}{9}+\dots+\frac{1}{n^2}<2-\frac{1}{n}, n \geq 2, n \in N$. 8

(b) The coefficients of three consecutive terms in the expansion of $(1 + x)^n$ are in the ratio 6:33:110. Find n. 8

III. (a) Sum the series : $1-\frac{1}{4}+\frac{1.3}{4.8}-\frac{1.3.5}{4.8.12}+\dots\dots$ 8

(b) If in triangle ABC,

$$\frac{\tan A - \tan B}{\tan A + \tan B} = \frac{c-b}{c}, \text{ prove that } A = \frac{\pi}{3}.$$ 8

UNIT—II

IV. (a) State and prove De Moivre's theorem for integral index. 8

(b) Show that the roots of equation $(x - 1)^n = x^n$ are given by $\frac{1}{2}\left(1+i\cot\frac{k\pi}{n}\right)$. where k = 0, 1, 2, ..., n–1 (n being a +ve integer). 8

V. (a) Separate $\tan^{-1}(x + iy)$ into real and imaginary parts. 8

(b) If $\cos(\theta + i\varphi) = r(\cos\alpha + i\sin\alpha)$, then prove that $\varphi = \frac{1}{2}\log\frac{\sin(\theta-\alpha)}{\sin(\theta+\alpha)}$, $\alpha, \theta, \varphi, r \in \mathbf{R}$. 8

UNIT—III

VI. (a) Prove that :

$$\begin{vmatrix} a^3+1 & a^2 & a \\ b^3+1 & b^2 & b \\ c^3+1 & c^2 & c \end{vmatrix} = -(a-b)(b-c)(c-a)(1+abc).$$ 8

(b) Show that every Hermitian matrix A can be uniquely expressed as P + iQ, where P and Q are real symmetric and real skew-symmetric matrices respectively. 8

VII. (a) Check whether the vectors $x = (2, -1, 1)$, $y = (1, 2, 1)$, $z = (3, -4, 1)$ are linearly dependent or not. If so, find the relation between them. 8

(b) Find the rank of the matrix :

$$\begin{bmatrix} 2 & -1 & 0 & 4 \\ 1 & 3 & 5 & -3 \\ 3 & -5 & -5 & 11 \\ 6 & 4 & 10 & 2 \end{bmatrix}$$ by reducing it to normal form. 8

UNIT—IV

VIII. (a) Find the values of a and b for which the system of equations :

$$2x - 3y + 5z = 12,$$

$$3x + y + az = b,$$

$$x - 7y + 8z = 17$$

has (i) Unique solution (ii) Infinitely many solutions (iii) No solution. 8

(b) Solve completely the system of equations :

$$x + 3y + 2z - s - t = 0,$$

$$2x + 6y + 5z + s - t = 0,$$

$$5x + 15y + 12z + s - 3t = 0.$$ 8

IX. (a) State and prove Cayley Hamilton theorem. 8

(b) Find an invertible matrix P such that $P^{-1}AP$ is in diagonal form where :

$$A = \begin{bmatrix} 5 & -2 & 1 \\ 2 & 10 & -2 \\ 9 & 0 & 9 \end{bmatrix}$$ 8