(i) **Printed Pages: 3** **Roll No.** ....

(ii) **Questions : 8** **Sub. Code : 1 7 5 4 2**

Exam. Code : 0 0 0 6

B.A./B.Sc. (General) 6th Semester

(2055)

MATHEMATICS

Paper—II : Linear Algebra

Time Allowed : Three Hours] [Maximum Marks : 30

Note :—Attempt **FIVE** questions in all, by selecting at least **TWO** from each Unit.

UNIT—I

1. (a) Let a, b, c be fixed elements of a field F. Show that :

$$W = \{(x, y, z) | ax + by + cz = 0; x, y, z \in R\}$$

is a subspace of $R_3(F)$. 3

(b) Show that every finitely generated vector space has basis. 3

2. (a) Find the value of 'k' so that the set

$$S = \{(1, 1, 1), (1, 1, 0), (1, 0, k)\}$$

is L.I. over R. 3

(b) Let $M = \{(a, b, c, d) \mid b + c + d = 0\}$,

$$N = \{a, b, c, d | a + b = 0, c = 2d\}$$

be two subspaces of R^4 over R, then find basis and dimension of (i) M, (ii) N, (iii) $M \cap N$. 3

3. (a) Find T(x, y) where $T : R^2 \to R^3$ is defined as

$T(1, 2) = (3, -1, 5)$ and $T(0, 1) = (2, 1, -1)$. 2

(b) State and prove Rank and Nullity Theorem. 4

4. (a) Prove that L.T, $T : V \to W$ over a field F is non-singular iff it is one-one. 3

(b) Let $T : V_3(R) \to V_3(R)$ be defined as

$$T(x, y, z) = (3x, x - y, 2x + y + z),$$

prove that T is invertible and find T^{-1}. 3

UNIT—II

5. Let $T : R^3 \to R^2$ is L.T. defined as

$$T(x, y, z) = (3x + 2y - 4z, x - 5y + 3z)$$

Verify that :

$$[T : B_1, B_2] [v ; B_1] = [T(v) : B_2] \ \forall \ v \in R^3$$

where $B_1 = \{(1, 1, 1), (1, 1, 0), (1, 0, 0)\}$ is basis of R^3 and $B_2 = \{(1, 3), (2, 5)\}$ is basis of R^2. 6

6. (a) Define Transition matrix and show that if B_1 and B_2 are two ordered basis of a vector space V over field F then transition matrix from B_1 to B_2 is invertible. 4

(b) If X is eigen vector of a matrix A then show that it cannot corresponds to more than one eigen value of A. 2

7. (a) Prove that eigen values of Hermitian matrix are real. 3

(b) If $A = \begin{bmatrix} 5 & -6 & -6 \\ -1 & 4 & 2 \\ 3 & -6 & -4 \end{bmatrix}$, find characteristic and minimal polynomial of A. 3

8. (a) State and prove Cayley Hamilton Theorem. 3

(b) Find eigen values and eigen vectors of :

$$A = \begin{bmatrix} 8 & -8 & -2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}.$$ 3