(i) **Printed Pages: 3** **Roll No. ....**

(ii) **Questions : 8** **Sub. Code : 1 7 5 4 1**

Exam. Code : 0 0 0 6

B.A./B.Sc. (General) 6th Semester

(2055)

MATHEMATICS

Paper—I : Analysis–II

Time Allowed : Three Hours] **[Maximum Marks : 30**

Note :—Attempt **FIVE** questions in all, selecting at least **TWO** questions from each unit.

UNIT—I

1. (a) If $f : A \to \mathbb{R}$ is defined by $f(x, y) = 3 \ \forall \ (x, y) \in A$, where $A = \left\{(x, y) \middle| \begin{matrix} 1 \le x \le 2 \\ 3 \le y \le 4 \end{matrix}\right\}$. Evaluate $\iint\limits_A f(x, y)\, dxdy$ using definition of double integral.

   (b) Let $A = \{(x, y) \mid 2 \le x \le 3, 4 \le y \le 5\}$. Show that $56 \le \iint\limits_A (2x^2 + 3y^2)\, dxdy \le 93$. 3+3=6

2. (a) Find the area enclosed by the cardioid $r = a(1 + \cos\theta)$.

   (b) Find the volume of a cylinder with base radius 2 and height 7 using Triple integration. 3+3=6

3. (a) Evaluate $\int_0^1\left(\int_{4y}^4 e^{x^2}\,dx\right)dy$.

(b) Evaluate $\iiint \frac{dxdydz}{(x+y+z+1)^3}$ over the region $x \geq 0$, $y \geq 0$, $z \geq 0$, $x + y + z \leq 1$. 3+3=6

4. (a) Verify Gauss Divergence Theorem for vector $\vec{A} = x\hat{i} + y\hat{j} + z\hat{k}$ over the region bounded by $x^2 + y^2 + z^2 = a^2$.

(b) Evaluate by Green's Theorem in plane for :

$\oint_C [(\cos x \sin y - xy)\,dx + \sin x \cos y\,dy]$. 3+3=6

UNIT—II

5. (a) Does pointwise convergence imply uniform convergence ? Justify your answer.

(b) State and prove M_n-Test for sequence of functions.

3+3=6

6. (a) Test for uniform convergence and continuity of the sum function for all values of x of the series :

$$\sum_{n=1}^{\infty} \frac{x}{n(1+nx^2)}.$$

(b) Consider the series $\sum_{n=1}^{\infty} f_n(x)$ in [0, 1], where

$f_n(x) = \frac{1}{1+nx}$. Test the series for uniform convergence and term by term integration in [0, 1]. 3+3=6

7. (a) State and prove Abel's Theorem for power series.

(b) Express in Fourier series the function $f(x) = x$ in $0 < x < 2\pi$. 3+3=6

8. (a) Express $\sin x$ as cosine series when $0 < x < \pi$.

(b) Obtain the Fourier series for $\sqrt{1 - \cos x}$ in $-\pi < x < \pi$. 3+3=6