(i) Printed Pages : 3 Roll No. ....

(ii) Questions : 8 **Sub. Code :** | 1 | 7 | 3 | 4 | 1 |

Exam. Code : | 0 | 0 | 0 | 4 |

B.A./B.Sc. (General) 4th Semester

(2055)

MATHEMATICS

Paper—I : Advanced Calculus—II

Time Allowed : Three Hours] **[Maximum Marks : 30**

Note :— Attempt **FIVE** questions in all, selecting at least **TWO** questions from each Unit.

UNIT—I

1. (a) By definition, show that the sequence $\left\langle \frac{2n-1}{n} \right\rangle$ converges to 2. 3

   (b) State and prove Squeeze principle. 3

2. (a) Prove that $\lim_{n\to\infty}\left(\frac{1}{(n+1)^{4/3}}+\frac{1}{(n+2)^{4/3}}+....+\frac{1}{(2n)^{4/3}}\right)=0$ 3

   (b) Show that the sequence $< a_n >$ where

   $a_n = 1+\frac{1}{3}+\frac{1}{3^2}+.....+\frac{1}{3^n}$ is convergent. 3

3. (a) State and prove Cauchy's General principle of convergence. 3

(b) Show that the sequence $< a_n >$, where

$$a_n = 1 + \frac{1}{3} + \frac{1}{5} + \ldots\ldots + \frac{1}{2n-1}$$ is not convergent. 3

4. (a) Show that the function f defined by

$$f(x) = \begin{cases} 1 & \text{if } x \text{ is rational} \\ -1 & \text{if } x \text{ is irrational} \end{cases}$$

is disontinuous everywhere. 3

(b) Show that $f(x) = \sin x$ is uniformly continuous on $\left[0, \frac{\pi}{2}\right]$.

3

UNIT–II

5. (a) Show that $\sum_{n=1}^{\infty} \frac{1}{n(n+1)} = 1$. 3

(b) Using integral test, discuss the convergence or divergence of the series $\sum_{n=1}^{\infty} \frac{1}{n^p}$, $p > 0$ 3

6. (a) Test for convergence or divergence of the series

$$\frac{2^1}{1^2}x + \frac{3^2}{2^3}x^2 + \frac{4^3}{3^4}x^3 + \ldots\ldots, \quad x > 0$$ 3

(b) Illustrate by a suitable example that Cauchy's root test is better than D'Alembert's ratio test. 3

7. (a) Examine the convergence or divergence of the series :

$$1+\frac{\alpha+1}{\beta+1}+\frac{(\alpha+1)(2\alpha+1)}{(\beta+1)(2\beta+1)}+.....$$ 3

(b) Show that the following series is convergent :

$$\frac{1}{\log 2}-\frac{1}{\log 3}+\frac{1}{\log 4}-\frac{1}{\log 5}+.....$$ 3

8. (a) Prove that series $x-\frac{x^3}{3}+\frac{x^5}{5}-\frac{x^7}{7}+.....$ is convergent for $-1 \le x \le 1$. 3

(b) Prove that if $\sum_{n=1}^{\infty} a_n$ convergence absolutely, then $\sum_{n=1}^{\infty}\left(\frac{n+1}{n}\right) a_n$ also converges absolutely. 3