Exam.Code:5002
Sub. Code: 10437

2055

Bachelor of Arts (FYUP) Second Semester
Statistics
Paper: Probability Theory

Time allowed: 3 Hours **Max. Marks: 60**

NOTE: *Attempt five questions in all, including Question No. I which is compulsory and selecting one question from each Unit. Simple non-programmable calculator is allowed.*

x-x-x

1. Answer the following:

(i) If for two events A and B, $P(A \cap B) = 0.5$, $P(A^c \cap B^c) = 0.5$ and $2P(A) = P(B) = \alpha$ (a constant). Then find the value of α.

(ii) Prove that probability of impossible event is zero.

(iii) State multiplicative law of probability.

(iv) State total probability theorem.

(v) If X and Y are independent variables, the prove that $E(X + Y) = E(X) + E(Y)$, provided all the expectation exists.

(vi) Suppose that the random variable X has moment generating function (mgf) M_X. Let $Y = \alpha X + \beta$. Then find the moment generating function of Y.

(vii) Define probability density function and probability mass function.

(viii) Let X be a random variable with probability density function (p.d.f.) as given below:

X	0	1	2	3
$P(X)$	1/3	1/2	1/24	1/8

Find the expected value of $Y = (X - 1)^2$.

(8×1.5)

Unit-I

2. (a) Describe random experiment, sample space, equally likely, mutually exclusive and exhaustive events.

(b) In a random arrangement of the letters of the word 'MATHEMATICS', find the probability that all the vowels come together. (08+04)

P.T.O.

3. (a) Explain the concept of classical, empirical and axiomatic probability. Discuss about the classical probability as a special case of axiomatic probability.

(b) A committee of 4 people is to be appointed from 3 officers of the production department, 4 officers of the purchase department, 2 officers of the sale department and 1 chartered accountant. Find the probability of forming the committee in the following manner: (a) There must be one from each category, (b) It should have at least one from the purchase department, (c) The chartered accountant must be in the committee. **(08+04)**

Unit-II

4. (a) State and prove addition theorem of probability. Also discuss its generalized form in case of more than two events.

(b) A box contains 6 red, 4 white and 5 black balls. A person draws 4 balls from the box at random. Find the probability that among the balls drawn there is at least one ball of each colour. **(08+04)**

5. (a) State and prove Bayes' theorem.

(b) The chance that 'doctor A' will diagnose a 'disease X' correctly is 60%. The chances of a patient will die by his treatment after correct diagnosis is 40% and the chance of death by wrong diagnosis is 70%. A patient of 'doctor A', who has 'disease X', died. What is the chance that his disease was diagnosed correctly? **(08+04)**

Unit-III

6. (a) What do you mean by marginal and conditional distributions? The following table represents the joint probability distribution of the discrete random variables

Y/X	1	2	3
1	1/2	1/6	0
2	0	1/9	1/5
3	1/18	1/4	2/15

(i) Evaluate marginal distribution of X.

(ii) Evaluate the conditional distribution of Y given $X = 2$.

(b) Two discrete random variables X and Y have $P(X = 0, Y = 0) = 2/9$; $P(X = 0, Y = 1) = 1/9$ $P(X = 1, Y = 0) = 1/9$; $P(X = 1, Y = 1) = 5/9$.

Examine whether X and Y are independent. **(06+06)**

7. (a) Suppose that X is a continuous random variable whose probability density function is given by

$$f(x) = \begin{cases} c(4x - 2x^2), 0 < x < 2 \\ 0, otherwise \end{cases}$$

(i) What is the value of c? (ii) Find $P(X > 1)$.

(b) Define random variable and its distribution function (or cumulative distribution function). Write the important properties of distribution function. (06+06)

Unit-IV

8. (a) Define the characteristic function of a random variable and its properties.

(b) Prove that the variance of X can be regarded as consisting of two parts, expectation of the conditional variance and variance of the conditional expectation. Symbolically, we have

$$Var(X) = E[Var(X|Y)] + Var[E(X|Y)]$$ (06+06)

9. (a) Define variance of a random variable and its properties.

(b) Write a note on probability generating function. (06+06)

x-x-x