(i)   Printed Pages: 4

(ii) Questions   : 9

Bachelor of Arts (FYUP) 2nd Semester

(2055)

MATHEMATICS

Paper : Calculus–I

Time Allowed : Three Hours]   [Maximum Marks : 90

Note :— Attempt **FIVE** questions in all. Select at least **ONE** question from each unit. Q. No. **1** is compulsory.

1.   (a)   If $a \geq 0$, $b \geq 0$, then show that $\dfrac{a+b}{2} \geq \sqrt{ab}$, with equality holds iff $a = b$.   3

     (b)   Evaluate $\lim\limits_{x \to 0} \dfrac{\sqrt{1+x}-1}{x}$.   3

     (c)   Find $\dfrac{dy}{dx}$, where $y = x \cosh x - \sinh x$.   3

     (d)   Find third derivative of $\log \sqrt{3x+4}$ w.r.t. x.   3

(e) Verify Rolle's theorem for $f(x) = x^2 - 4x + 3$, $1 \leq x \leq 3$. 3

(f) Evaluate $\lim_{x \to 0} \left(\dfrac{e^x - e^{-x}}{\sin x} \right)$. 3

UNIT–I

2. (a) Prove that $\sqrt{2}$ is irrational number. 6

(b) State and prove Archimedean property of real numbers. 6

(c) Find g.l.b. and l.u.b. of the set

$S = \{3 \sin x + 4 \cos x \mid x \in \mathbb{R}\}$. 6

3. (a) If $f(x) = \begin{cases} x & \text{when} \quad x \text{ is rational} \\ -x & \text{when} \quad x \text{ is irrational} \end{cases}$

show that $\lim_{x \to 0} f(x) = 0$. 9

(b) State and prove Squeeze principle. 9

UNIT–II

4. (a) Show that if f is continuous at $x = a$ then $|f|$ is also continuous at $x = a$ but the converse is false. 9

(b) Examine the continuity of the function

$f(x) = \begin{cases} \dfrac{|x-a|}{x-a} & \text{if} \quad x \neq a \\ 1 & \text{if} \quad x = a \end{cases}$ at $x = a$. 9

5. (a) If $f(x) = \begin{cases} 3ax+b & \text{if } x>1 \\ 11 & \text{if } x=1 \\ 5ax-2b & \text{if } x<1 \end{cases}$ is continuous at $x = 1$.

Find the values of a and b. 9

(b) Show that $f(x) = \begin{cases} x\cos\dfrac{1}{x} & , \quad x \neq 0 \\ 0 & , \quad x = 0 \end{cases}$ is continuous for

all real x. 9

UNIT–III

6. (a) Find $\dfrac{d}{dx}\{x^x + (\sinh x)^x\}$. 9

(b) If $y = \sin(m \sin^{-1} x)$ find $y_n(0)$. 9

7. (a) Find the values of a and b so that

$$\lim_{x \to 0} \frac{x(1+a\cos x) - b\sin x}{x^3}$$ exists and it equals to one.

9

(b) Find the points on the curve $y = 5x^2 - 2x^3$ at which the tangent is parallel to the line $y = 4x + 5$. 5

(c) Use differentials to approximate $\sqrt{25.3}$. 4

UNIT–IV

8. (a) State and prove L.M.V. theorem. 9

 (b) Find first three non zero terms in the Maclaurin's expansion of sin x. 9

9. (a) State and prove Cauchy mean value theorem. 9

 (b) Find position of real zeros of f'(x) where

$$f(x) = x(x - 1)(x - 2)(x - 3).$$ 9