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B.A./B.Sc. (General) 5th Semester

(2122)

MATHEMATICS

Paper–II (Modern Algebra)

Time Allowed : Three Hours] [Maximum Marks : 30

Note :— Attempt **five** questions in all, choosing at least **two** questions from each Unit. All questions carry equal marks.

UNIT–I

1. (a) Find the orders of elements of Quaternions group Q_8. Is Q_8 cyclic ? Justify.

   (b) Let G be a semi-group and a, b $\in$ G. Prove that G is a group if and only if both the cancellation laws hold in G. 2,4

2. (a) If H and K are two subgroups of a group G, then prove that HK is subgroup of G iff HK = KH.

   (b) Let G be a group and a, b $\in$ G. Is O(ab) = O(a).O(b), in general ? Justify. 4,2

3. (a) Define index of a subgroup H in a group G. Prove that O(G) = O(H)[G : H].

   (b) Let G be a group such that G/Z(G) is cyclic, where Z(G) is centre of the group G, then prove that G is abelian. 3,3

4. (a) Prove that there are only two groups of order 6.

(b) Compute $p^{-1}qp$ where $p = (1, 3, 5)(2, 4)$ and $q = (1, 4, 2, 5)(6, 3)$. 3,3

UNIT–II

5. (a) Define division ring. Prove that every division ring is a simple ring.

(b) Let E be the ring of even integers. Prove that $<4>$ is a maximal ideal in E. 3,3

6. (a) Prove that a commutative ring with unity is a field iff it does not have any proper ideal.

(b) Prove that in the ring of integers Z, the ideal $<m> = mZ = \{mn : n \in Z\}$ is a prime ideal if and only if m is prime number. 3,3

7. (a) State and prove fundamental theorem of ring homomorphism.

(b) Is the field **R** of real numbers isomorphic to the field **C** of complex numbers ? Justify. 4,2

8. (a) If R is an integral domain, then prove that R[x] is also integral domain.

(b) What are the units of the polynomial ring $Z_7[x]$?

3,3