(i) Printed Pages : 7 Roll No. ....

(ii) Questions : 9 **Sub. Code : 0 9 0 5**

Exam. Code : 0 1 2 2

Bachelor of Computer Applications 3rd Year

1046

DISCRETE MATHEMATICS

Paper : BCA-27

Time Allowed : Three Hours] **[Maximum Marks : 90**

Note :– Attempt **five** questions in all, including Question No. 9 (Section E) which is compulsory and selecting **one** question each from Sections A-D.

SECTION—A

1. (a) Partition A = {0, 1, 2, 3, 4, 5} with the minsets generated by B_1 = {0, 2, 4}, B_2 = {1, 5}. How many different subsets of A can you generate from B_1 and B_2 ?

   (b) Each of the following defines a relation on the set N of positive integers :

   $$R : x > y; \;\; T = x + 4y = 10 \text{ for all } x, y \in N$$

   Determine which of the relations are :

   (i) reflexive

   (ii) symmetric

   (iii) transitive.

(c) Which of the following functions are injections, surjections, or bijections on R :

(i) $f(x) = -2x$

(ii) $g(x) = x^2 - 1$. 6,6,6

2. (a) Solve the recurrence relation for Fibonacci numbers given by :

$f_n = f_{n-1} + f_{n-2}$, subject to $f_1 = f_2 = 1$, where $n \geq 3$.

(b) Find the generating function from the recurrence relation given by :

$s(k) = 6s(k-1) - 5s(k-2)$, where $s(0) = 1$, $s(1) = 2$.

9,9

SECTION—B

3. (a) If V = {1, 2, 3, 4, 5} and E = {(1, 2), (2, 3), (3, 3), (3, 4), (4, 5)}. Find the number of edges and size of graph G = (V, E).

(b) Differentiate between paths and circuits.

(c) Show that the maximum number of edges in a graph with n vertices and no multiple edges are $\frac{n(n-1)}{2}$.

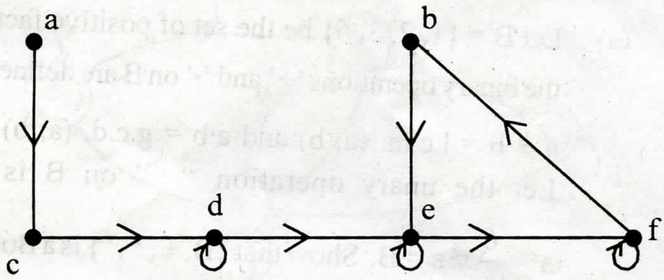
(d) Draw graph in which :

(i) no edge is cut edge

(ii) every edge is a cut edge

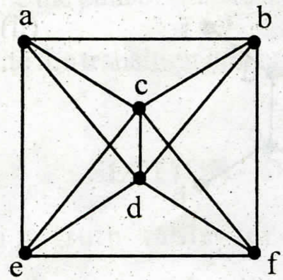
(iii) only one cut vertex. 5,5,5,3

4. (a) The directed graph corresponding to the adjacency matrix M_A is shown in figure given below :



Find the adjacency matrix M_A.

(b) Consider the graph G shown in the following figure :



(i) Find the Euler's path if it exists.

(ii) Is the graph G Eulerian ?

(iii) Is the graph Hamiltonian ?

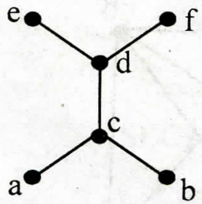
(c) How many edges are there in an undirected graph with 10 vertices each of degree 6 ? 6,9,3

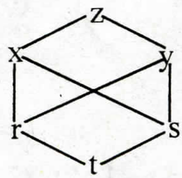
SECTION—C

5. (a) Let $B = \{1, 2, 3, 6\}$ be the set of positive factors of 6. Let the binary operations '+' and '·' on B are defined as follows :

$a + b = \text{l.c.m.}(a, b)$ and $a \cdot b = \text{g.c.d.}(a, b)$ $\forall$ $a, b \in B$. Let the unary operation " ′ " on B is defined by $a' = \frac{6}{a}$ $\forall$ $a \in B$. Show that $\{B, +, \cdot, '\}$ is a Boolean algebra.

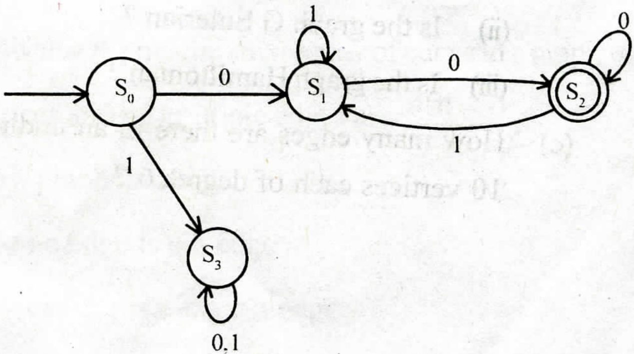
(b) Determine whether the posets shown in the figure given below are lattice or not.

(i) 

(ii) 

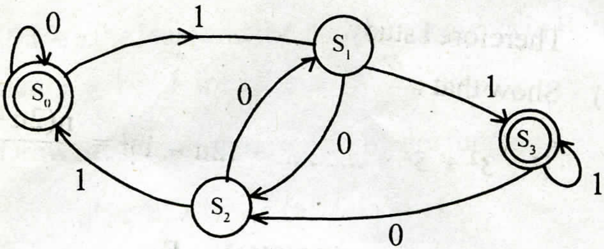
12,6

6. (a) Describe the following state machine :



Also determine language.

(b) The state diagram of the finite automaton is given below :



(i) Write the accept states.

(ii) Does it accept empty string ?

(iii) Write the path of states machine follow on input 0111.

(iv) Write the transition table of finite state machine.

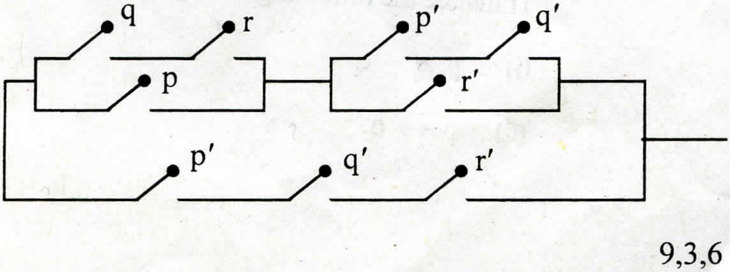
9,9

SECTION—D

7. (a) Construct truth table for the Boolean function $f(x_1, x_2, x_3) = (x_1 \wedge x_2) \vee (x_1 \vee (\overline{x}_2 \wedge x_3))$.

(b) Express $E(x, y, z) = x(y'z)'$ in its complete sum of products form.

(c) Use Boolean algebra to simplify the switching circuit.



9,3,6

8. (a) Check the validity of the argument. If I work, I cannot study. Either I work or pass mathematics. I passed mathematics. Therefore I study.

(b) Show that :

$$1^2 + 3^2 + 5^2 + \ldots\ldots\ldots + (2n-1)^2 = \frac{n(2n-1)(2n+1)}{3}.$$

12,6

SECTION—E

9. (a) If $A = \{+, -\}$ and $B = \{00, 01, 10, 11\}$. Find $A \times B$.

(b) If $f(x) = \frac{x}{x+1}$, $g(x) = \frac{1}{x-1}$, find $(f \circ g)(x)$.

(c) Find the order of the recurrence relation $s(k) = s(k/2) + 9$, $k \geq 0$.

(d) If P(n) is the statement "12n + 8 is a multiple of 5", then check P(3) and P(6).

(e) In the Boolean algebra $(B, +, \cdot, ')$, show that for $a, b \in B$, $a = 0$ if $a + b = 0$.

(f) Let p denote the statement, "The weather is nice" and q denote the statement "We have a picnic".

Translate the following in English :

(i) $p \wedge \sim q$

(ii) $p \leftrightarrow q$.

(g) Is there a graph with 8 vertices of degree 2, 2, 3, 6, 5, 7, 8, 4 ? Justify your answer.

(h) 0, 1 are the least and the greatest elements in B, then show that $a \vee 1 = 1$ for all $a \in B$.

(i) Draw the circuit represented by the function :

$$(a' \wedge b' \wedge c) \vee (a' \wedge b \wedge c).$$

2×9=18